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ASYMPTOTIC SOLUTION OF THE PROBLEM OF HEAT TRANSFER BETWEEN A PLATE AND AN UNBOUNDED UNIFORM FLUID FLOW†

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The three leading terms of the asymptotic expansion of the solution of the problem of convective heat transfer between a thin plate of finite length and arbitrary surface temperature and an unbounded uniform fluid flow are obtained analytically for low Péclet and Prandtl numbers. © 2001 Elsevier Science Ltd. All rights reserved.

Despite the fact that, in practice, the case when the Reynolds and Prandtl numbers satisfy the conditions $\mathrm{Re} \gg 1$, $\mathrm{Pr} \ll 1$ is rarely observed [1], the problem of convective heat transfer between a plate of finite length and an unbounded uniform fluid flow is still of interest as a problem in mathematical physics, which is often encountered in different areas of applied mechanics, for example, in contact problems of the theory of elasticity [2], in problems of the freezing of seeping soils [3], etc. When a constant temperature is maintained on the plate surface, there is an analytical solution, which has been obtained by several researchers [2, 4–7] and which is a series in Mathieu functions. This fact makes it difficult to use both in numerical and qualitative analysis. In this paper, unlike those mentioned, a representation of the solution in terms of Mathieu functions is not used, and the asymptotic form is constructed directly using the apparatus of integral equations and analytical functions of a complex variable.

1. FORMULATION OF THE PROBLEM

The steady convective heat transfer between a thin plate of finite length and an unbounded uniform fluid flow (see Fig. 1) is described by the following system of equations [8]

$$\begin{aligned} &\mathrm{Pe}\,\partial\theta/\partial x = \Delta\theta, \quad x, y \in D \\ &\theta = 0, \quad x^2 + y^2 \to \infty; \quad \theta = f(x), \quad x \in \Gamma = [-1, 1] \end{aligned} \tag{1.1}$$

where x and y are dimensionless Cartesian coordinates (referred to the half-length of the plate), D is the flow region (the exterior of the section $\Gamma = [-1, 1]$), θ is the temperature, measured from the temperature at infinity, $f(x)$ is the specified temperature distribution on the plate, and Pe is the Péclet number, constructed from the half-length of the plate, the free-stream velocity and the thermal diffusivity of the fluid.

Note that the problem is symmetrical about the $y = 0$ axis, and hence the condition $\partial\theta/\partial y = 0$ is satisfied on the sections $|x| > 1$ of this axis.

We will assume that the function $f(x)$ can be approximated by the series

$$f(x) = \sum_{k=0}^{N} c_k T_k(x) \tag{1.2}$$

where T_k are Chebyshev polynomials of the first kind [9]. In view of the linearity of system (1.1) it is sufficient to consider the case $f(x) = T_k(x)$, $k \geq 0$.

Using the method of boundary integral equations, problem (1.1) can be reduced to the following integral equation [10]

$$-\frac{1}{\pi}\int_{-1}^{1} \mu(\xi)\exp\left[\frac{\mathrm{Pe}}{2}(x-\xi)\right]K_0\left(\frac{\mathrm{Pe}}{2}|x-\xi|\right)d\xi = T_k(x), \quad x \in \Gamma, \quad k \geq 0 \tag{1.3}$$

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where

$$\mu(x) = \partial\theta/\partial y, \quad x \in \Gamma, \quad y = +0$$

After solving it the overall heat flux to the plate $2Q$ and the temperature $\theta(x, y)$ in the region D can be determined from the formulae

$$Q = \int_{-1}^{1} \mu(\xi) d\xi \tag{1.4}$$

$$\theta(x, y) = -\frac{1}{\pi} \int_{-1}^{1} \mu(\xi) \exp\left[\frac{\mathrm{Pe}}{2}(x - \xi)\right] K_0\left(\frac{\mathrm{Pe}}{2} r\right) d\xi, \quad r = \sqrt{(x - \xi)^2 + y^2} \tag{1.5}$$

It is convenient to introduce the following notation

$$\nu(\xi) = \mu(\xi) \exp\left(-\frac{\mathrm{Pe}}{2}\xi\right), \quad \Theta(x, y) = -\theta(x; y) \exp\left(-\frac{\mathrm{Pe}}{2}x\right) \tag{1.6}$$

We can then obtain the following integral representation for the function $\Theta(x, y)$ from (1.5)

$$\Theta(x, y) = \frac{1}{\pi} \int_{-1}^{1} \nu(\xi) K_0\left(\frac{\mathrm{Pe}}{2} r\right) d\xi, \quad x, y \in D \tag{1.7}$$

while from condition (1.3) we obtain the boundary equation

$$\Theta(x, y)|_\Gamma = -T_k(x) \exp\left(-\frac{\mathrm{Pe}}{2}x\right), \quad k \geqslant 0 \tag{1.8}$$

2. THE CASE OF LOW PÉCLET NUMBER

When $\mathrm{Pe} \ll 1$ and $x^2 + y^2 \sim 1$ the kernel of integral operator (1.7) and the right-hand side of Eq. (1.8) can be expanded asymptotically. We will have [9]

$$K_0\left(\frac{\mathrm{Pe}}{2} r\right) = (m - \ln r) + \frac{\mathrm{Pe}^2 r^2}{16}[m + 1 - \ln r] + O(\mathrm{Pe}^4), \quad m = \ln\frac{4}{\mathrm{Pe}} - C \tag{2.1}$$

$$T_k(x) \exp\left(-\frac{\mathrm{Pe}}{2}x\right) = T_k(x) - \frac{\mathrm{Pe}}{4}[T_{k+1}(x) + T_{|k-1|}(x)] +$$

$$+ \frac{\mathrm{Pe}^2}{32}[T_{k+2}(x) + 2T_k(x) + T_{|k-2|}(x)] + O(\mathrm{Pe}^3) \tag{2.2}$$

where C is Euler's constant.

To carry out an asymptotic analysis we will assume that $\nu(\xi)$, Q and $\Theta(x, y)$ can be represented in the form of asymptotic series in powers of Pe

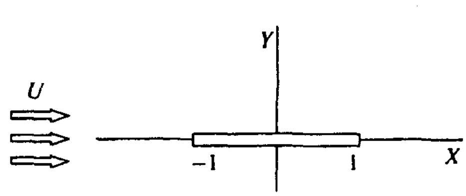

Fig. 1

$$\nu(\xi)=\nu_0(\xi)+\mathrm{Pe}\,\nu_1(\xi)+\mathrm{Pe}^2\,\nu_2(\xi)+O(\mathrm{Pe}^3) \tag{2.3}$$

$$Q=Q_0+\mathrm{Pe}Q_1+\mathrm{Pe}^2Q_2+O(\mathrm{Pe}^3) \tag{2.4}$$

$$\Theta(x,y)=\Theta_0(x,y)+\mathrm{Pe}\,\Theta_1(x,y)+\mathrm{Pe}^2\,\Theta_2(x,y)+O(\mathrm{Pe}^3) \tag{2.5}$$

The relation $m(\mathrm{Pe})$ in expansion (2.1) of the kernel of the integral equation indicates that terms of the order of $\mathrm{Pe}^i\ln^j\mathrm{Pe}$ must be present in these representations. The form of (2.3)–(2.5) denotes that, when constructing the solution formally, m is assumed to be a constant quantity and in fact $\nu_i(\xi)$, Q_i, $\Theta_i(x,y)$ will depend on Pe via m. This procedure enables us to reduce and adjust the calculations, since the coefficients of Pe^i obtained using it depend only slightly (logarithmically) on Pe and do not spoil the asymptotic form of expansions (2.3)–(2.5).

Substituting (2.3) and (2.4) into (1.4) and (1.6) we obtain

$$Q_i=q_i+\frac{\bar\delta_{i0}}{2}\int_{-1}^{1}\nu_{i-1}(\xi)\xi d\xi+\frac{\delta_{i2}}{8}\int_{-1}^{1}\nu_{i-2}(\xi)\xi^2d\xi,\quad i=0,1,2; \tag{2.6}$$

$$q_i=\int_{-1}^{1}\nu_i(\xi)d\xi \tag{2.7}$$

(where δ_{ij} is the Kronecker delta, $\bar\delta_{ij}=1-\delta_{ij}$).

We substitute (2.1), (2.3) and (2.5) into representation (1.7) of the function $\Theta(x,y)$ and separate it into orders. We obtain the integral representations

$$\Theta_i(x,y)=\frac{mq_i}{\pi}-\frac{1}{\pi}\int_{-1}^{1}\nu_i(\xi)\ln r d\xi+\delta_{i2}F(x,y),\quad i=0,1,2 \tag{2.8}$$

where

$$F(x,y)=\frac{1}{16\pi}\int_{-1}^{1}\nu_0(\xi)r^2[m+1-\ln r]d\xi \tag{2.9}$$

We further substitute expressions (2.2) and (2.5) into Eq. (1.8) and we also separate the relation obtained into orders. We obtain boundary conditions for the functions $\Theta_i(x,y)$

$$\Theta_i(x,y)|_\Gamma=-\delta_{i0}T_k(x)+\frac{\delta_{i1}}{4}[T_{k+1}(x)+T_{|k-1|}(x)]-\frac{\delta_{i2}}{32}[T_{k+2}(x)+2T_k(x)+T_{|k-2|}(x)],$$
$$i=0,1,2;\quad k\geqslant 0 \tag{2.10}$$

Hence, problem (1.1) for low Pe reduces to a series of problems for the logarithmic potentials (2.7)–(2.10), covering the three leading terms of the asymptotic form.

3. SOLUTION OF BOUNDARY INTEGRAL EQUATIONS OF THE TYPE (2.8) AND (2.10) IN TERMS OF ANALYTICAL FUNCTIONS

As is well known [11], a constructive analysis of boundary-value problems of this kind can be carried out using the eigenvalue relation

$$-\frac{1}{\pi}\int_{-1}^{1}\frac{T_n(\xi)}{\sqrt{1-\xi^2}}\ln|x-\xi|d\xi=\delta_{n0}\ln 2+\frac{\bar\delta_{n0}}{n}T_n(x),\quad x\in[-1,1] \tag{3.1}$$

where $n\geqslant 0$, but the solution in the form of logarithmic potentials is inconvenient to use. We will express relation (3.1) in terms of analytical functions of the complex variable $z=x+iy$.

We will introduce the following function

$$\mathcal{P}(z)=z-\sqrt{z^2-1}$$

In order to ensure uniqueness, we will agree to mean by the root that branch of it which is identical with the root that is arithmetic on the real axis when $x > 1$. The function $\mathcal{P}(z)$ is the inverse of the Zhukovskii function [12] and maps the region D onto the unit circle $|\mathcal{P}| \leqslant 1$. On the section Γ we obviously have

$$\mathcal{P}(z)|_\Gamma = \exp(i \arccos x) \tag{3.2}$$

whence it follows [9] that

$$\mathcal{P}^n(z)|_\Gamma = T_n(x) + iU_n(x), \qquad n \geqslant 1$$

where $U_n(x)$ are Chebyshev polynomials of the second kind.

The case $n = 0$ in relation (3.1) corresponds to the problem of conductive heat flux from an infinitely distant source to a plate having a constant temperature. In the plane of the complex variable $\mathcal{P}$ this corresponds to the problem of the conductive heat flux from a source situated at the point $\mathcal{P} = 0$ to the contour $|\mathcal{P}| = 1$ having a constant temperature. The complex thermal potential of this problem, apart from an unimportant additive constant, will be equal to $\lambda \ln \mathcal{P}$, where λ is a parameter which defines the total heat flux to the plate. Taking this into account, and also expression (3.2), we can obtain the required form of relation (3.1) in terms of the analytical function $\Omega(z)$, such that

$$\mathrm{Re}\,\Omega(z) = -\frac{1}{\pi}\int_{-1}^{1} \frac{\nu^*(\xi)}{\sqrt{1-\xi^2}} \ln r\, d\xi \tag{3.3}$$

In fact, it follows from relation (3.1), that a density $\nu^*(\zeta)$ of the form

$$\nu^*(\xi) = T_n(\xi), \qquad n \geqslant 0 \tag{3.4}$$

corresponds to the function

$$\Omega(z) = \delta_{n0} \ln[2\mathcal{P}(z)] + \frac{\bar{\delta}_{n0}}{n} \mathcal{P}^n(z), \quad n \geqslant 0 \tag{3.5}$$

Here we have used the fact that when $n = 0$ the parameter $\lambda = 1$. This follows from a comparison of the behaviour of the function $\mathrm{Re}\Omega(z)$ in the form (3.3) and (3.5) for $z = x \to \infty$, taking into account the equation [13]

$$\int_{-1}^{1} \frac{\nu^*(\xi)}{\sqrt{1-\xi^2}} d\xi = \delta_{n0}\pi$$

A similar check of the behaviour of the function $\mathrm{Re}\Omega(z)$ at infinity can also be carried out for $n \geqslant 1$. The following assertion follows from the above relations.

Assertion. The solution of the boundary equation

$$\mathrm{Re}\,\Omega(z)|_\Gamma = T_n(x), \quad n \geqslant 0$$

for a function of the form (3.3) can be represented by the expression

$$\Omega(z) = \delta_{n0}\left[1 + \frac{\ln \mathcal{P}(z)}{\ln 2}\right] + \bar{\delta}_{n0} \mathcal{P}^n(z), \quad n \geqslant 0 \tag{3.6}$$

where

$$\int_{-1}^{1} \frac{\nu^*(\xi)}{\sqrt{1-\xi^2}} d\xi = \delta_{n0} \frac{\pi}{\ln 2} \tag{3.7}$$

4. CONSTRUCTION OF THE SOLUTION

We will construct the solution of problem (2.7)–(2.10) in terms of analytical functions $\Omega_i(z)$

$$\operatorname{Re}\Omega_i(z) = -\frac{1}{\pi}\int_{-1}^{1}\frac{v_i^*(\xi)}{\sqrt{1-\xi^2}}\ln r d\xi, \quad v_i^*(\xi) = v_i(\xi)\sqrt{1-\xi^2} \tag{4.1}$$

connected with $\Theta_i(x, y)$ by the formula

$$\Theta_i(x,y) = \operatorname{Re}\Omega_i(z) + \frac{mq_i}{\pi} + \delta_{i2}F(x,y) \tag{4.2}$$

which follows from representations (2.8) and (4.1).

By (4.2) and (2.10) the following relation is satisfied on the boundary Γ

$$\operatorname{Re}\Omega_i(z)|_\Gamma = \delta_{i0}\left[T_k(x) - \frac{mq_0}{\pi}\right] - \delta_{i1}\left\{\frac{1}{4}[T_{k+1}(x) + T_{|k-1|}(x)] + \frac{mq_1}{\pi}\right\} +$$

$$+\delta_{i2}\left\{\frac{1}{32}[T_{k+2}(x) + 2T_k(x) + T_{|k-2|}(x)] - \frac{mq_2}{\pi} - F(x,y)|_\Gamma\right\} \tag{4.3}$$

Then, for the function $\Omega_0(z)$, in view of the assertion from Section 3, the following representation holds

$$\Omega_0(z) = \left(\delta_{k0} - \frac{mq_0}{\pi}\right)\left[1 + \frac{\ln\mathcal{P}(z)}{\ln 2}\right] + \bar{\delta}_{k0}\mathcal{P}^k(z)$$

Relations (2.7) and (3.7) enable us to determine the constant q_0 and the final form of the function $\Omega_0(z)$

$$\Omega_0(z) = \frac{q_0}{\pi}\ln[2\mathcal{P}(z)] + \bar{\delta}_{k0}\mathcal{P}^k(z), \quad q_0 = \frac{\delta_{k0}\pi}{m + \ln 2} \tag{4.4}$$

We similarly obtain

$$\Omega_1(z) = \frac{q_1}{\pi}\ln[2\mathcal{P}(z)] - \frac{1}{4}[\bar{\delta}_{k1}\mathcal{P}^{|k-1|}(z) + \mathcal{P}^{k+1}(z)], \quad q_1 = -\frac{\delta_{k1}\pi}{4(m+\ln 2)} \tag{4.5}$$

The problem for $\Omega_2(z)$ is somewhat more complex in view of the presence of the non-harmonic contribution of $F(x, y)$ in formula (4.2) and the corresponding contribution in boundary condition (4.3). We will express $F(x, y)$ in terms of the functions $\mathcal{P}(z)$.

First, using the eigenvalue relation (3.1) we obtain, from boundary equation (4.3), (4.1), the form of the function $\nu_0^*(\zeta)$

$$v_0^*(\xi) = \frac{q_0}{\pi} + kT_k(\xi) \tag{4.6}$$

Substituting this into (2.10), taking relations (3.3)–(3.4) and (4.4) into account, we obtain

$$F(x,y) = \frac{m+1}{16}\left(\frac{\delta_{k2}}{2} - x\delta_{k1} + \omega\frac{q_0}{\pi}\right) + \frac{\delta_{k2} - 2x}{32}\ln[2|\mathcal{P}(z)|] + \frac{\omega}{16}\operatorname{Re}\Omega_0(z) -$$

$$-\frac{x}{16}\operatorname{Re}\left\{\frac{2q_0}{\pi}\mathcal{P}(z) + k\left[\frac{\mathcal{P}^{k+1}(z)}{k+1} + \bar{\delta}_{k1}\frac{\mathcal{P}^{|k-1|}(z)}{|k-1|}\right]\right\} + \operatorname{Re}\Omega^*(z) \tag{4.7}$$

$$\omega = x^2 + y^2 + \frac{1}{2}, \quad \Omega^*(z) = \frac{q_0}{64\pi}\mathcal{P}^2(z) + \frac{k}{64}\left[\frac{\mathcal{P}^{k+2}(z)}{k+2} + \bar{\delta}_{k2}\frac{\mathcal{P}^{|k-2|}(z)}{|k-2|}\right]$$

Obviously $\Omega^*(z)$ is a function of the type (3.3), it consists of several terms of the form (3.6), and none of them makes a contribution to integral (3.7). It turns out that for this it is sufficient that it should make no contribution to the final expression for $\Theta_2(x,y)$. Hence, in our further calculations we will not specify the form of the terms related to this.

Now, from relations (3.2), (4.3) and the form of the function $F(x, y)$ obtained, we can write the boundary condition for the function $\Omega_2(z)$

$$\operatorname{Re}\Omega_2(z)|_\Gamma = \frac{mq_2}{\pi} - \frac{m+1+\ln 2}{16}\left[\frac{\delta_{k2}}{2} - \delta_{k1}T_1(x)\right] - \operatorname{Re}\Omega^*(z)|_\Gamma +$$
$$+\frac{T_{k+2}(x)+T_{|k-2|}(x)}{64} + \frac{k}{32}\left[\frac{T_{k+2}(x)+T_k(x)}{k+1} + \bar{\delta}_{k1}\frac{T_{|k-2|}(x)+T_k(x)}{|k-1|}\right]$$

Applying the assertion from Section 3 to this, we obtain

$$\Omega_2(z) = \frac{\delta_{k1}}{16}(m+\ln 2+1)\mathcal{P}(z) + \frac{q_2}{\pi}\ln[2\mathcal{P}(z)] + \frac{q_0\mathcal{P}^2(z)}{32\pi} - \Omega^*(z) + \Phi(z)$$
$$\Phi(z) = \frac{k}{32}\left[\frac{\mathcal{P}^{k+2}(z)+\mathcal{P}^k(z)}{k+1} + \bar{\delta}_{k1}\frac{\bar{\delta}_{k2}\mathcal{P}^{|k-2|}(z)+\mathcal{P}^k(z)}{|k-1|}\right] + \frac{\mathcal{P}^{k+2}(z)+\bar{\delta}_{k2}\mathcal{P}^{|k-2|}(z)}{64},$$
$$q_2 = \frac{\delta_{k2}\pi}{64}\left(\frac{3}{m+\ln 2} - 2\right) \tag{4.8}$$

Using expressions (4.4)–(4.8) for the functions $\Omega_i(z)$, $F(x, y)$ and the quantities q_i, from (4.2) we obtain

$$\Theta_0(x,y) = -\delta_{k0} - \operatorname{Re}\left[\frac{q_0}{\pi}\ln\mathcal{P}(z) + \bar{\delta}_{k0}\mathcal{P}^k(z)\right]$$
$$\Theta_1(x,y) = \frac{\delta_{k1}}{4} - \operatorname{Re}\left[\frac{q_1}{\pi}\ln\mathcal{P}(z) - \frac{1}{4}\mathcal{P}^{k+1}(z) - \frac{\bar{\delta}_{k1}}{4}\mathcal{P}^{|k-1|}(z)\right] \tag{4.9}$$
$$\Theta_2(x,y) = -\frac{\omega}{16}\operatorname{Re}[\ln\mathcal{P}(z)+\mathcal{P}^k(z)] - \sum_{j=0}^{2}\delta_{jk}\operatorname{Re}\Phi_j(z) -$$
$$-\operatorname{Re}\left[\Phi(z) - \frac{kx}{16}\left(\frac{\mathcal{P}^{k+1}(z)}{k+1} + \bar{\delta}_{k1}\frac{\mathcal{P}^{|k-1|}(z)}{|k-1|}\right)\right]$$

where

$$\Phi_0(z) = \frac{2\omega+\mathcal{P}^2(z)-4x\mathcal{P}(z)}{32(m+\ln 2)}$$
$$\Phi_1(z) = \frac{m+\ln 2+1}{16}[\mathcal{P}(z)-x] - \frac{x}{16}\ln\mathcal{P}(z) \tag{4.10}$$
$$\Phi_2(z) = \frac{5}{64} + \frac{3\ln\mathcal{P}(z)}{64(m+\ln 2)}$$

As a check it can be shown that the representations obtained for the functions $\Theta_i(x,y)$ satisfy boundary conditions (2.10).

Relations (4.9) and (4.10), taking into account relations (1.6) and (2.5) between the functions $\Theta_i(x,y)$ and $\theta(x,y)$, give the required analytical representation of the asymptotic expansion of problem (1.1) for low Pe, retaining the three leading terms of the asymptotic form.

Note that the functions $\Theta_i(x,y)$ depend on Pe via the parameter m (see relation (2.10)), which only occurs in the quantities q_0, q_1 and the functions $\Phi_j(z)$. It is obvious that when $k > 2$ the functions $\Theta_i(x,y)$ will generally not depend on Pe.

To calculate the overall heat flux to the plate using relations (2.4)–(2.6), it is also necessary to obtain the function $v_1^*(\xi)$. This can be done by applying relations (3.4) and (3.5) to the function $\Omega_1(z)$, the form of which is known. We will have

$$v_1^*(\xi)=\frac{q_1}{\pi}-\frac{k+1}{4}T_{k+1}(\xi)-\bar{\delta}_{k1}\frac{|k-1|}{4}T_{|k-1|}(\xi) \tag{4.11}$$

After substituting the second expressions from (4.4), (4.5), (4.8) and expressions (4.6) and (4.11) into relation (3.6) we can obtain the quantity Q_i, and, of course, also the total heat flux $2Q$ to the plate, apart from terms $\sim \mathrm{Pe}^3$

$$\frac{Q}{\pi}\approx -\frac{1}{m+\ln 2}\left[\delta_{k0}-\mathrm{Pe}\frac{\delta_{k1}}{4}+\mathrm{Pe}^2\frac{4\delta_{k0}+3\delta_{k2}}{64}\right]-\mathrm{Pe}\frac{\delta_{k1}}{4}+\mathrm{Pe}^2\frac{4\delta_{k0}+\delta_{k2}}{32}$$

Hence, in particular, it follows that the order of the quantity Q with respect to Pe is determined by the first non-zero term of expansion (1.2).

5. COMPARISON WITH NUMERICAL EXPERIMENT AND THE RESULTS OF OTHER RESEARCHES

We compared the asymptotic formulae obtained with numerical calculations. Equation (1.3) was solved numerically with $k = 0$ using the method proposed previously in [14], with subsequent establishment of the overall heat flux to the plate, using Eq. (1.4), and the temperature, using Eq. (1.5). It turned out that the asymptotic relation $Q(\mathrm{Pe})$ approximates the numerical relationship quite well up to a value of $\mathrm{Pe} = 1$, at which the error reaches $\approx 4\%$ (omitting second-order terms increases the error to $\approx 13\%$). The situation is somewhat worse for the temperature, since it is not an integral characteristic of the problem but a differential one. A comparison of the temperatures on the contour $|z| = 2$, $\operatorname{Re} z > 0$ showed good agreement up to a value of $\mathrm{Pe} = 0.5$, for which the error reached $\approx 5\%$.

Note that when $f(x) = \text{const}$ the principal term of the expansion of Q with respect to Pe agrees with that obtained previously in [15]. A comparison with the results obtained in [6] shows a difference both in the principal term – an additional factor of 2 occurs in [6], and in the second-order term with respect to Pe (the first-order term is zero). The good agreement with the results of numerical calculations, as mentioned above, suggests that the asymptotic form obtained is correct. For comparison we note that the error of the asymptotic formula in [6] for $\mathrm{Pe} = 1$ is of the order of 100%. Here the difference between the numerical value of Q derived in [6] and that obtained using the method proposed earlier in [14] does not exceed 2%.

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